Cooper pair sizes in ^{11}Li and in superfluid nuclei: a puzzle?

K. Hagino[1], H. Sagawa[2] and P. Schuck[3,4]

[1] Department of Physics, Tohoku University, Sendai, 980-8578, Japan

E-mail: `hagino@nucl.phys.tohoku.ac.jp`

[2] Center of Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8560, Japan

[3] Institut de Physique Nucléaire, CNRS, UMR8608, Orsay, F-91406, France

[4] Université Paris-Sud, Orsay, F-91505, France

Abstract. We point out a strong influence of the pairing force on the size of the two neutron Cooper pair in ^{11}Li, and to a lesser extent also in ^{6}He. It seems that these are quite unique situations, since Cooper pair sizes of stable superfluid nuclei are very little influenced by the intensity of pairing, as recently reported. We explore the difference between ^{11}Li and heavier superfulid nuclei, and discuss reasons for the exceptional situation in ^{11}Li.

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It is well known that pairing correlations enhance cross sections for two-neutron transfer reactions (see Refs. [1, 2] for recent reviews on pair transfer). A few theoretical calculations have revealed that a spatially confined neutron pair (*i.e.,* dineutron or Cooper pair) exists on the nuclear surface for two particles (or two holes) around a core nucleus with shell closure [3, 4, 5, 6, 7, 8]. The enhancement of two-neutron transfer cross sections has been attributed to this effect.

Probably it is Hansen and Jonson who exploited the idea of dineutron correlation explicitly for exotic nuclei for the first time. They proposed the dineutron cluster model and successfully analysed the matter radius of ^{11}Li [9]. They also predicted a large Coulomb dissociation cross section of the ^{11}Li nucleus.

Recently, the dineutron correlation has attracted much attention in connection with neutron-rich nuclei, partly due to the new measurement for the Coulomb dissociation of ^{11}Li [10], which have shown a strong indication of the existence of a correlated dineutron in ^{11}Li. In fact, many theoretical discussions on the dineutron correlation have been taking place in recent years, not only in the $2n$ halo nuclei, ^{11}Li and ^{6}He [11, 12, 13, 14, 15], but also in medium-heavy neutron-rich nuclei [16, 17, 18, 19, 20, 21] as well as in infinite neutron matter [22, 23, 24]. These calculations have shown that

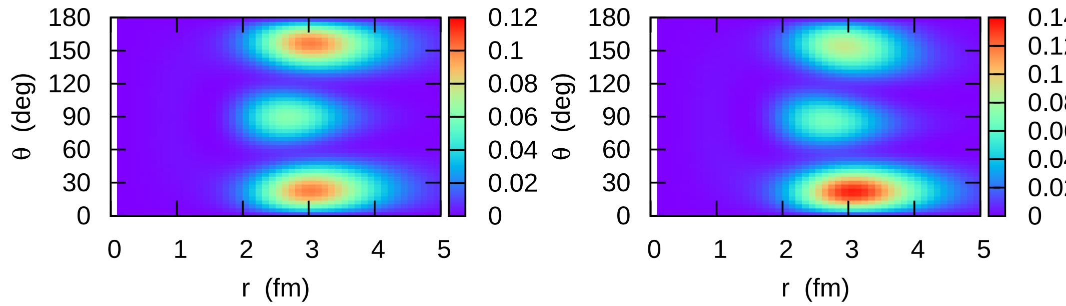

Figure 1. The two particle density for ^{18}O nucleus obtained with a three-body model of ^{16}O+n+n as a function of the distance of the neutron r_1=r_2=r from the core nucleus and the opening angle between the valence neutrons, θ. The left panel shows the two particle density obtained by including only the bound 1d$_{5/2}$, 2s$_{1/2}$, and 1d$_{3/2}$ single-particle levels in the three-body model calculation, while the right panel is obtained by including single particle levels up to $e_{\rm max}$ =30 MeV and $l_{\rm max}$ = 7.

the dineutron correlation is enhanced in neutron-rich nuclei, although it exists also in stable nuclei.

It is easy to understand why dineutron correlations become more significant if the two neutrons are close to the continuum threshold. For this purpose, we show in Fig. 1 the result of a three-body model calculation for ^{18}O nucleus, ^{16}O+n+n, in which the valence neutrons interact with each other via a density-dependent contact interaction [25] (that is, the so called surface type pairing interaction) ‡. For the n-^{16}O potential, we use the same Woods-Saxon potential as in Ref. [4], which has three bound levels, 1d$_{5/2}$, 2s$_{1/2}$, and 1d$_{3/2}$, above the N = 8 shell closure. The left panel shows the two particle density, $\rho(r, r, \theta)$, as a function of the neutron-core distance r and the opening angle between the two neutrons, θ, obtained by including only the bound levels in the three-body model calculation. Here, we set $r_1 = r_2 = r$ for presentation purposes. The figure shows symmetric two peaks at $\theta \sim 0$ and $\theta \sim \pi$. The right panel, on the other hand, is obtained by including single particle levels up to $e_{\rm max}$ =30 MeV and $l_{\rm max}$ = 7. In this case, the two peaks become strongly asymmetric, the peak around $\theta \sim 0$ being much more enhanced as compared to the other peak. This is nothing more than the manifestation of dineutron correlations which we shall discuss in this article. To obtain the spatially compact dineutron, it has been recognized that the mixing of single particle levels of opposite parities by the pairing interaction plays an essential role [7, 20]. That is, the pairing interaction mixes the bound positive parity levels with a lot of continuum levels with negative parity. As the Fermi energy becomes smaller, the continuum states play a decisive role and the admixture of opposite parity states takes place more significantly in neutron-rich nuclei, leading to strong dineutron correlations.

‡ We have also used Gogny equivalent contact forces [26] and have reached the same conclusion on Cooper pair sizes.

Although the dineutron correlation in the ^{11}Li nucleus has been discussed for two decades since the publication of Hansen and Jonson, many questions have yet to be answered, especially also for dineutron correlations in heavier neutron-rich nuclei. In this article, we discuss such open problems concerning the pairing properties in neutron-rich nuclei from the point of view of dineutron correlations. In particular, we shall discuss the local size of dineutrons.

Many properties of superfluidity are related to the size of Cooper pairs relative to the mean interparticle distance and to the size of the system [27]. In the BCS approximation, it is well known that the size of a Cooper pair in infinite matter is characterized by the coherence length given by Pippard's relation

$$\xi = \frac{\hbar^2 k_F}{m^* \pi \Delta}, \tag{1}$$

where k_F is the Fermi momentum, m^* is the effective mass, and Δ is the pairing gap [27, 28]. In atomic nuclei, the coherence length estimated with Eq. (1), averaged over the volume in a local density approximation (LDA) procedure, is of the size of a larger nucleus.

The density dependence of the coherence length has been investigated recently by Matsuo for infinite nuclear and neutron matter [22]. It was shown that the coherence length of a nn pair first shrinks as density decreases from the normal density, and then expands again after taking a minimum at around $\rho/\rho_0 \sim 0.1$. See also Refs. [29] for a similar behaviour for a np pair, which has a bound state at $\rho = 0$.

In Ref. [13], we have shown that the root mean square distance between the valence neutrons in ^{11}Li exhibits a qualitatively similar density dependence as the pair moves from the center of the nucleus to the nuclear surface and to free space §. Moreover, subsequent Hartree-Fock-Bogoliubov calculations with the Gogny interaction have confirmed that the size shrinking behaviour exists generically also in medium-heavy nuclei with strong superfluidity, both in stable and neutron-rich nuclei[20]. See also Ref. [31].

A question has arisen concerning what causes a small Cooper pair on the nuclear surface. As we have mentioned, in finite nuclei, the coherence length estimated in infinite nuclear matter, Eq. (1), is about the nuclear size.

Very recently, the size effect has been studied for the ^{120}Sn nucleus [32]. It has turned out that the relative importance between pairing and size effects is totally opposite between ^{11}Li and ^{120}Sn. That is, the coherence length for a Cooper pair in ^{120}Sn is affected very little by the pairing interaction and takes a minimum of about 2 fm on the surface even in a situation of negligible pairing gap. This has been seen for other superfluid nuclei in various mass regions as well [32].

Indeed, for the ^{11}Li and ^{6}He nuclei, the pairing effect seems to dominate over the size effect. This is demonstrated in Fig. 2, where we show the local coherence length of the Cooper pair in ^{11}Li and ^{6}He as a function of the nuclear radius R obtained with

§ The size of deuteron in ^{6}Li also behaves similarly [30]. However, it is not clear whether the mechanism is the same (see below).

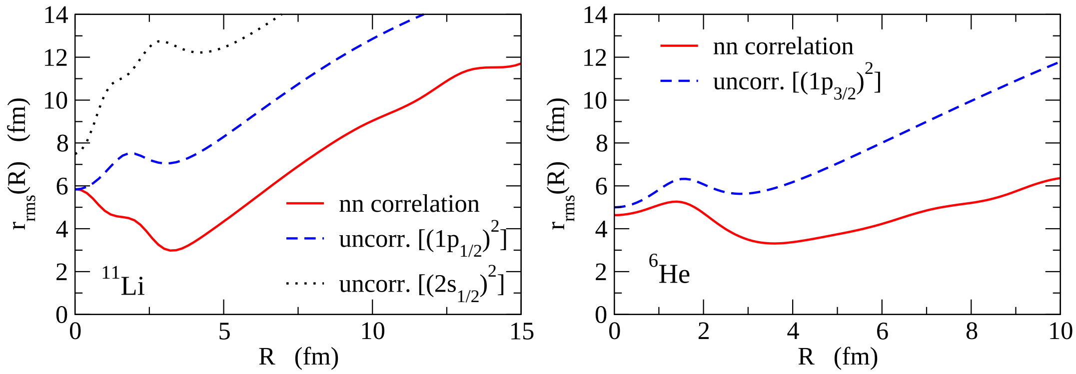

Figure 2. (the left panel) The root mean square distance $r_{\rm rms}$ for the neutron Cooper pair in ^{11}Li as a function of the nuclear radius R. The solid line shows the result of three-body model calculation with density dependent contact pairing force, while the dashed and the dotted lines are obtained by switching off the neutron-neutron interaction and assuming $[(1\mathrm{p}_{1/2})^2]$ and $[(2\mathrm{s}_{1/2})^2]$ configurations, respectively. For the uncorrelated cases, the single-particle potential is adjusted so that the corresponding single-particle energy is -0.15 MeV. (the right panel)The same as the left panel, but for ^{6}He nucleus. The dashed line corresponds to the uncorrelated pair in the $1\mathrm{p}_{3/2}$ orbital at $e = -0.49$ MeV.

and without the nn interaction. For the uncorrelated calculations for ^{11}Li, we consider both the $[(1\mathrm{p}_{1/2})^2]$ and $[(2\mathrm{s}_{1/2})^2]$ configurations and adjust the single-particle potential so that the corresponding single-particle energy is -0.15 MeV. For ^{6}He, we consider an uncorrelated pair in the $1\mathrm{p}_{3/2}$ orbital at -0.49 MeV. One can see that, in the non-interacting case, the Cooper pair continuously expands, as it gets farther away from the center of the nucleus. In marked contrast, in the interacting case it becomes smaller going from inside to the surface before expanding again into the free space configuration.

A reason why ^{6}He and ^{11}Li behave differently from ^{120}Sn with respect to the coherence length may be that the neutron pairs in ^{6}He and ^{11}Li are bound much more weakly than in ^{120}Sn. We will argue that the main reason is that the dominant components in the ground state wave function in ^{6}He and ^{11}Li are low angular momentum states with zero or one node. This may be inferred from the fact that the rms distance for uncorrelated $(2\mathrm{s}_{1/2})^2$ and $(1\mathrm{p}_{1/2})^2$ pairs in ^{11}Li, as well as an uncorrelated $(1\mathrm{p}_{3/2})^2$ pair in ^{6}He, take a pronounced minimum when the binding is deep, as shown in Fig. 3. In this case, the behaviour of the rms distance indeed resembles the one for the correlated pair shown in Fig. 2. On the other hand, for an uncorrelated $(1\mathrm{d}_{5/2})^2$ pair in ^{11}Li, the rms distance shows a clear minimum even when the binding is extremely weak (see the middle lower panel in Fig. 3). This can be explained by the fact that a halo wave function is only connected with s- and p-waves in the zero energy limit because of the small centrifugal barrier, as has been studied in Refs. [33, 34]. For an uncorrelated pair in ^{176}Sn, on the other hand, the rms distance takes a minimum both for $(1\mathrm{i}_{13/2})^2$ and $(3\mathrm{p}_{1/2})^2$ configurations even for a small binding energy, although the rms distance of the latter does not get lower than 10 fm for $e = -0.032$ MeV and the dependence

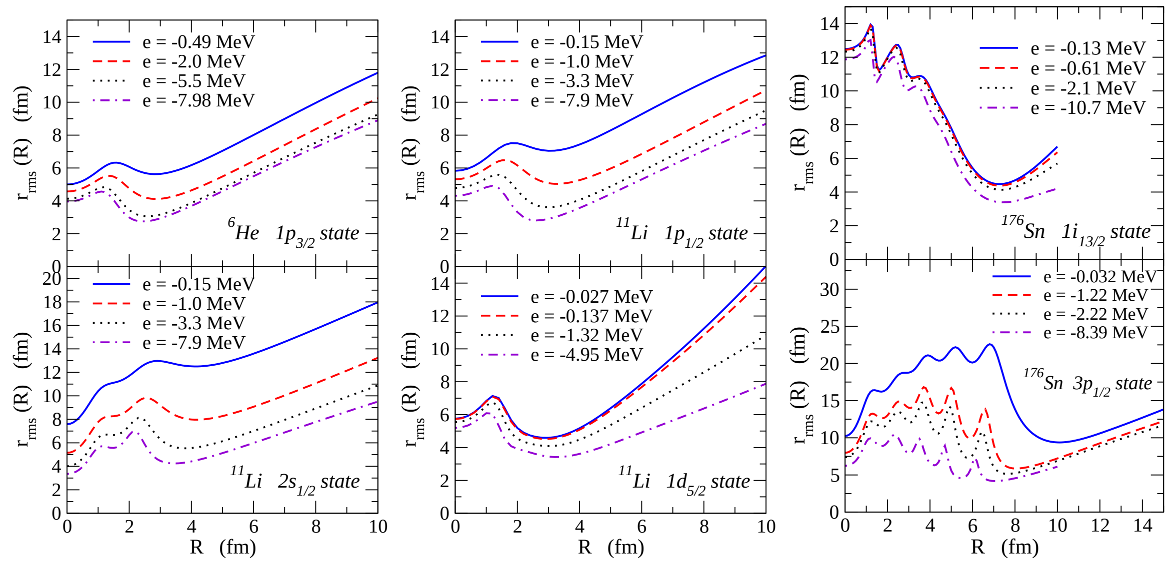

Figure 3. The root mean square distance $r_{\rm rms}$ for the *uncorrelated* neutron Cooper pair in ^{6}He, ^{11}Li, and ^{176}Sn nuclei for various single-particle angular momenta and energies as indicated in the figure.

on the single-particle energy is much stronger in $(3\mathrm{p}_{1/2})^2$ than in $(1\mathrm{i}_{13/2})^2$ (see the right panels in Fig. 3). For the $(3\mathrm{p}_{1/2})^2$ configuration, the higher nodal structure may cause the difference between ^{11}Li and ^{176}Sn. At any rate, ^{11}Li and ^{6}He seem to be very unique cases with respect to the influence of the pairing interaction on the value of the local rms distance of the Cooper pair. It would be interesting to find further exceptional examples of this kind in the nuclear chart. In general, one must conclude that besides rare cases such as ^{11}Li and to a lesser extent ^{6}He, the small rms radius of Cooper pairs in the surface of nuclei is essentially provoked by the size dependence of the single particle wave functions and not by pairing. The influence of the latter drops out from a compensation in numerator and denominator of the normalised two body wave function [32, 35].

Even though in general pairing does not seem to play an important role in the coherence length of Cooper pairs in standard superfluid nuclei, one should not forget its important influence on other quantities, as *e.g.* the strong reduction of the moment of inertia from its classical value. We have already seen in Fig. 1 the strong influence of pairing interaction also on the density distribution of ^{18}O. In Fig. 4, we demonstrate it again for ^{11}Li in a different way in connection to the size of Cooper pair. That is, Fig. 4 shows the two dimensional plot for the square of the radial part of two-particle wave function, $\Psi(R, r)^2$, for ^{11}Li multiplied by r^2R^2. The solid line denotes the local coherence length shown in Figs. 2 and 3. The left, the middle, and the right panels correspond to the correlated pair, the uncorrelated $(1p_{1/2})^2$ pair with the single-particle energy of $e = -0.15$ MeV, and the uncorrelated pair with $e = -7.9$ MeV, respectively. For the uncorrelated pair, there are two peaks with almost the same height. One of the peaks is

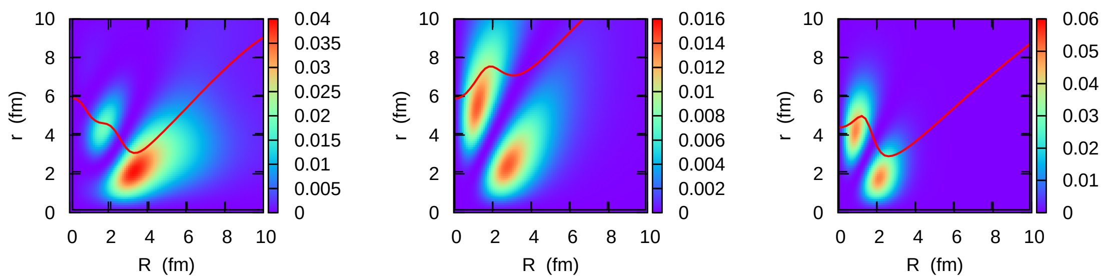

Figure 4. The square of the radial part of two-particle wave function for ^{11}Li. The multiplicative factor of r^2R^2 is taken into account. The left, middle, and right panels correspond to the the correlated pair, the uncorrelated $(1p_{1/2})^2$ pair with the single-particle energy of $e = -0.15$ MeV, and the uncorrelated $(1p_{1/2})^2$ pair with $e = -7.9$ MeV, respectively. The local coherence length as a function of R shown in Figs. 2 and 3 is also plotted by the solid line.

located at small r, and this peak is monitored when R is increased from $R = 0$, leading to the minimum in the local coherence length for the uncorrelated pair with $e = -7.9$ MeV. For the uncorrelated pair with $e = -0.15$ MeV, both peaks contribute to the local coherence length at around $R \sim 3$ fm, and the behaviour of rms distance appears more complex. For the correlated pair, on the other hand, the peak with larger r is much smaller than the peak with smaller r, due to the strong pairing effect. Therefore, it seems to be a general effect, not depending on a particular nucleus. For comparisons, we also show the square of the two-particle wave functions for the s and d waves in ^{11}Li and p wave in ^{176}Sn in Figs. 5,6 and 7, respectively.

The size of Cooper pairs in resonantly interacting atomic gases has been measured using radio-frequency spectroscopy [36]. In nuclear physics, a two-neutron transfer reaction has been considered to be a good probe of pairing correlation, although it would be extremely difficult to measure the size of Cooper pairs directly.

In summary, we argued that the very small size of Cooper pairs of about 2 fm, which have recently been pointed out in several works to exist on the surface of finite nuclei, may be of radically different origin in various nuclei. Actually it seems that in most cases this small size of Cooper pairs on the nuclear surface has nothing to do with enhanced pairing correlations on the surface of nuclei but rather is a consequence of the

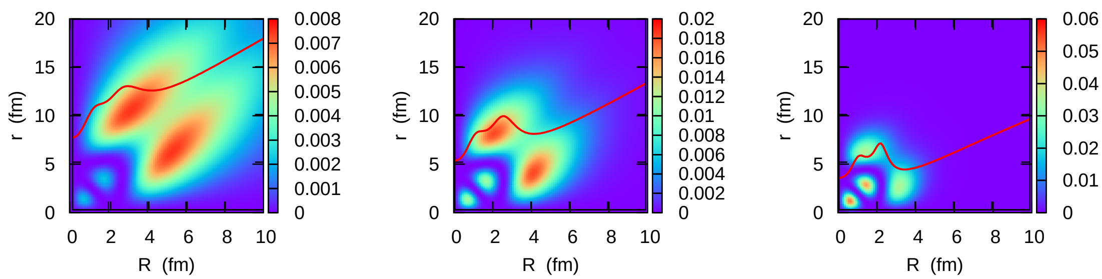

Figure 5. Same as Fig. 4, but for an uncorrelated $(2s_{1/2})^2$ pair in ^{11}Li with $e = -0.15$ MeV (the left panel), $e = -1.0$ MeV (the middle panel), and $e = -7.9$ MeV (the right panel).

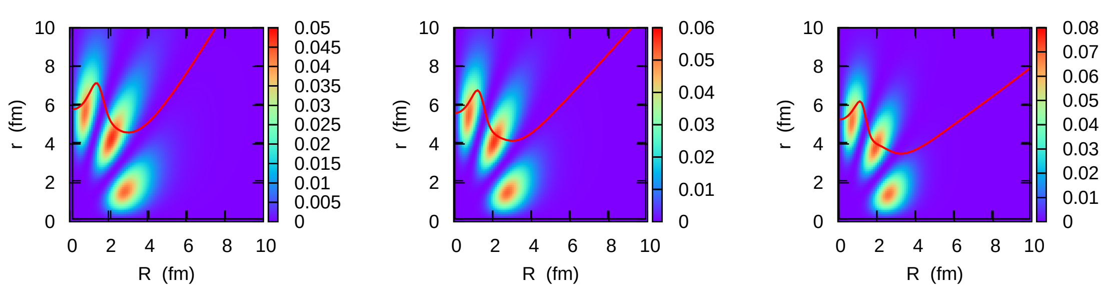

Figure 6. Same as Fig. 4, but for an uncorrelated $(1d_{5/2})^2$ pair in ^{11}Li with $e = -0.137$ MeV (the left panel), $e = -1.32$ MeV (the middle panel), and $e = -4.95$ MeV (the right panel).

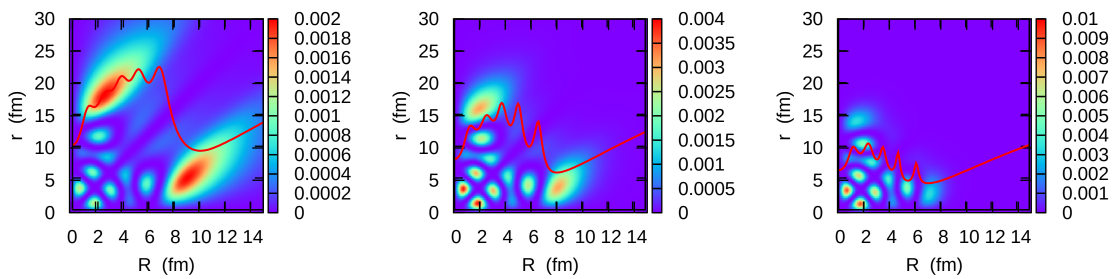

Figure 7. Same as Fig. 4, but for an uncorrelated $(3p_{1/2})^2$ pair in ^{176}Sn with $e = -0.032$ MeV (the left panel), $e = -1.22$ MeV (the middle panel), and $e = -8.39$ MeV (the right panel).

finiteness of the single-particle wave functions [32]. On the contrary, and this seems to be a quite unique and exceptional situation, in ^{11}Li and to a lesser extent also in ^{6}He, the Cooper pair size seems to be strongly influenced by the pairing interaction. This stems from the fact that the single-particle wave functions mostly involved are $2s$ and $1p$ states with very small binding. In that case ($l \leq 1$), the centrifugal barrier is very low and the single-particle wave functions can spread out very far (to infinity in the zero energy limit [34]), and make a halo structure [33]. It would be important to exploit this unique situation of ^{11}Li and study the influence and structure of the effective nn force in much detail. Analysis of an ongoing experimental work in ^{11}Li [37] is therefore most important. Investigations of whether further similar cases to ^{11}Li and ^{6}He exist in the nuclear chart for heavier nuclei may be very relevant.

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